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SPECTRUM OF ELEMENTARY PARTICLES IN A MODEL OF HADRON SUPERSYMMETRY.

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Abstract

We investigate a spectrum of the low-energy composite particles with the quantum numbers $J^p = 0^{\pm}, \frac{1}{2}^{\pm}$ in a $SU_F(3)$ model of hadron supersymmetry. We derive the mass spectrum of two, three and four-quark states and determine all free parameters of a theory, including the masses of quarks and diquarks.

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In the recent years, the supersymmetry (SUSY) approach was applied to the low energy physics of hadrons [1]. It was shown that some hadronic states form a representation space of the $N = 1$ SUSY algebra. But unsolved problem is how this algebra acts on the states of conventional QCD. In some papers [2], a possible way to solve this problem inside $SU_F(3)$ flavour quark model was given, if we identify diquarks (colour triplet states of two quarks)as the scalar quark states, which are the superpartners of ordinary quarks.

In this paper, we give a formulation of a model of hadron SUSY and derive an appropriate low energy model of hadrons, where SUSY is spontaneously broken. We compute the masses of hadrons, using experimental data of pseudoscalar two-quark mesons,scalar four-quark mesons, and barions [3]. By these data, we determine all free parameters of the theory and predict some new states in a scalar meson sector.

Consider a model of hadron SUSY. It contains the following superfields (SF's):

$$\Phi_{\pm\alpha i} = a_{\pm\alpha i} + \sqrt{2}\theta q_{\pm\alpha i} + \theta\theta\phi_{\pm\alpha i},$$
$$\bar{\Phi}_{\pm\alpha i} = a^*_{\pm\alpha i} + \sqrt{2}\bar{\theta}\bar{q}_{\pm\alpha i} + \bar{\theta}\bar{\theta}\phi^*_{\pm\alpha i}.$$

Here

$$q_{\alpha i} = \begin{pmatrix} q_{+\alpha i} \\ \bar{q}_{-\alpha i} \end{pmatrix}$$

is a quark dirac spinor field; $a_{\pm\alpha i}, a^*_{\pm\alpha i}$ are scalar quark fields which can be identified with the diquark fields; $\phi_{\pm\alpha i}$, $\phi^*_{\pm\alpha i}$ are auxiliary fields; α, i are indices of $SU_c(3)$ and $SU_F(3)$ respectively. All these fields are the components of the chiral SF's

$$\Phi_{\alpha i} = \begin{pmatrix} \Phi_{+\alpha i} \\ \bar{\Phi}_{-\alpha i} \end{pmatrix}.$$

We use notations of ref.[4]. Let us include some additional external Higgs SF's $\chi, \bar{\chi}$ and gauge SF's u, v in an adjoint representation of the chiral groups $SU_{RF}(3)$ and $SU_{LF}(3)$, respectively. These SF's are needed for soft SUSY breaking, they also give masses for quarks and diquarks. In the component form we have

$$u = -v = \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}D;$$

$$\chi = \frac{1}{2}(m + \theta\theta f), \bar{\chi} = \frac{1}{2}(m + \bar{\theta}\bar{\theta}f),$$

and D, f, m-are $SU_F(3)$ matrices of the form

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}.$$

The action of the theory is $S_0 =< \Phi, \mathbf{K_0}\Phi >$ (all indices are omitted), where we use a pairing in a chiral SF space $< \Phi_1, \Phi_2 >= \int d^4x d^2\theta \Phi_{+1}\Phi_{-2} + \int d^4x d^2\bar\theta \bar\Phi_{+2}\bar\Phi_{-1}$, instead of usual pairing for the dirac spinor fields $< \psi_1, \psi_2 >= \int d^4x \psi_{+1}\psi_{-2} + \int d^4x \bar\psi_{+2}\bar\psi_{-1}$. The superkinetic operator $\mathbf{K_0}$ is

$$\mathbf{K_0} = \begin{pmatrix} \chi & \bar D^2 e^{-v} \\ D^2 e^u & \bar\chi \end{pmatrix}$$

in a superfield form, and

$$\mathbf{K_0} = \begin{pmatrix} \frac{m}{2} & 1 & 0 & 0 & 0 & 0 \\ \partial^2 + \frac{D}{4} & 0 & 0 & \frac{f}{2} & 0 & 0 \\ \frac{f}{2} & 0 & 0 & \partial^2 + \frac{D}{4} & 0 & 0 \\ 0 & 0 & 1 & \frac{m}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m}{2} & i\hat\partial \\ 0 & 0 & 0 & 0 & i\hat{\bar\partial} & \frac{m}{2} \end{pmatrix}$$

in a component form; it is acting on a component field graded space

$$\Phi = \begin{pmatrix} a_+ \\ \phi_+ \\ a_-^* \\ \phi_-^* \\ q_+ \\ \bar q_- \end{pmatrix}.$$

Now we introduce an effective low energy action in a usual way [5]. We consider a chiral transformation matrix $\widetilde\Omega$ which is similar to ordinary matrix $\Omega = e^{i\gamma_5\pi(x)}$ for the chiral field $\pi(x)$. Instead of $\pi(x)$ we have the SUSY extension:

$$\gamma = \pi + i\sigma + \sqrt{2}\theta b + \theta\theta F,$$
$$\bar\gamma = \pi - i\sigma + \sqrt{2}\bar\theta\bar b + \bar\theta\bar\theta F^*,$$

where $\gamma, \bar{\gamma}$ are in an adjoint representation of $SU_F(3)$, and π, σ, b, $\bar{b}$ are stand for pseudoscalar, scalar and spinor effective fields, respectively. So, for the supermatrix $\widetilde{\Omega}$ we have:

$$\widetilde{\Omega} = \begin{pmatrix} e^{i\gamma_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{i\gamma_2} & 0 & -iF^* & 0 & i\bar{b} \\ iF & 0 & e^{-i\gamma_2^*} & 0 & -ib & 0 \\ 0 & 0 & 0 & e^{-i\gamma_2^*} & 0 & 0 \\ ib & 0 & 0 & 0 & e^{i\gamma_1} & 0 \\ 0 & 0 & 0 & -i\bar{b} & 0 & e^{-i\gamma_1^*} \end{pmatrix}.$$

Here $\gamma_1 = \pi_1 + i\sigma_1, \gamma_1^* = \pi_1 - i\sigma_1$ are parameters of chiral transformation of the pure quark fields, so π_1 and σ_1 - are pseudoscalar and scalar two-quark effective states, as in ordinary QCD (formally $\gamma_1 = q\bar{q}$). Parameters $\gamma_2 = \pi_2 + i\sigma_2, \gamma_2^* = \pi_2 - i\sigma_2$, are acting on the pure bosonic diquark states, so π_2 and σ_2 can be identified as pseudoscalar and scalar four-quark states (formally $\gamma_2 = aa^* = qq\bar{q}\bar{q}$). And the fermionic parameters b, $\bar{b}$ stand for barion states ($b = a^*q = qqq$).

Then, as usual,

$$\Gamma_{eff}(\pi_1, \sigma_1, \pi_2, \sigma_2, b, \bar{b}) = \frac{1}{2} S\mathrm{Tr}\log \mathbf{K} = -\frac{1}{4}\int_{\frac{1}{\Lambda^2}}^{\frac{1}{\mu^2}} \frac{dt}{t} S\mathrm{Tr} e^{-t\mathbf{K}^2},$$

where $\mathbf{K} = \widetilde{\Omega}\mathbf{K_0}\widetilde{\Omega}$, and we have used the proper time formalizm for definition of $S\mathrm{Tr}\log\mathbf{K}$. After the computation of supertrace and all needed integrals in Tr,we obtain the desired quadratic part of Γ_{eff}:

$$\begin{aligned} \Gamma_{eff} \;=\; & 4\mathrm{tr} q^2 m^2 \int d^4x \pi_2 \partial^2 \pi_2 + 4\mathrm{tr}(p^2 + q^2m^2) \int d^4x (b\bar{\sigma}^\mu i\partial_\mu \bar{b}) + \\ & 2\mathrm{tr}(2p^2 + q^2(m^2 - D)) \int d^4x \sigma_2 \partial^2 \sigma_2 - \\ & \mathrm{tr}(4f^2q^2(1 + \frac{4m^2q^2}{p^2 + q^2m^2}) - 4D(p^2 + 2q^2m^2) - m^2(6p^2 + q^2m^2)) \times \\ & \int d^4x \sigma_2^2 - 2\mathrm{tr} q^2 m f \int d^4x (bb + \bar{b}\bar{b}) - \\ & 2\mathrm{tr}(2q^2m^2(-D) - m^2(2p^2 + q^2m^2)) \int d^4x \pi_2^2 + 4\mathrm{tr} p^2 \int d^4x \pi_1 \partial^2 \pi_1 + \\ & 2\mathrm{tr} q^2 m^2 \int d^4x \sigma_1 \partial^2 \sigma_1 - 2\mathrm{tr} m^2 (2p^2 + q^2m^2) \int d^4x \pi_1^2 - \\ & \mathrm{tr} m^2 (6p^2 + q^2m^2) \int d^4x \sigma_1^2. \end{aligned}$$

Here p^2, q^2-are functions of Λ^2, μ^2, and they determine the low-energy region of integration.

Γ_{eff} is independent of regularization procedure because it is expressed through the Seeley coefficients for the operator $\mathbf{K}$(for the review, see ref.[6]).

After determination of all parameters p^2, q^2, m, f, D by the data of meson and barion spectroscopy, we have the following table for the composite low-energy particles.

Two-quark meson states						
$J^p = 0^-$				$J^p = 0^+$		
mesons	experiment (MeV)	theory (MeV)		mesons	experiment (MeV)	theory (MeV)
$\pi^{\pm,0}$	140	140	$I = 1$	-	-	370
$K^{\pm,0}$	490	490	$I = \frac{1}{2}$	-	-	860
η_0	550	560	$I = 0$	f_0	975	970

Four-quark meson states						
$J^p = 0^-$				$J^p = 0^+$		
mesons	experiment (MeV)	theory (MeV)		mesons	experiment (MeV)	theory (MeV)
π	1300	1350	$I = 1$	a_0	980	980
-	-	1500	$I = \frac{1}{2}$	K_0^*	1430	1430
η	1440	1550	$I = 0$	f_0	1590	1550

For the sake of simplicity, all theoretical masses of the barion octet $J^p = \frac{1}{2}^{\pm}$ are equal to the mass of $\Sigma^{\pm}$: $m_{b,\bar{b}} = m_{\Sigma} = 1190 MeV$.

In our model, we use the octet of scalar mesons as pure four-quark states for definition of the matrix D, barion octet for determination of the matrix f, and the octet of pseudoscalar mesons as pure two-quark states for determination of the quark mass matrix m. Then, we predict the masses of pure two-quark scalar states and pure four-quark pseudoscalar states, as shown at the tables.

Finally, we get the quark masses

$$m_u = m_d = 95MeV, m_s = 240MeV,$$

and the diquark masses

$$m_u^d = m_d^d = (320, 1020)MeV, m_s^d = (540, 1070)Mev.$$

In this paper, we do not discuss a mixing between two, four-quark and glueball states in the sector of scalar mesons. It is related to the theories of non- $q\bar{q}$ states(for the review, see [7] and references therein). The next step of the investigation would be to take into account a mixing between such states in a real world of scalar mesons.

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